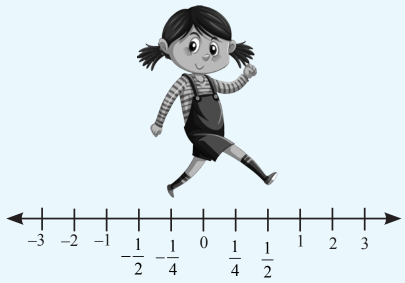
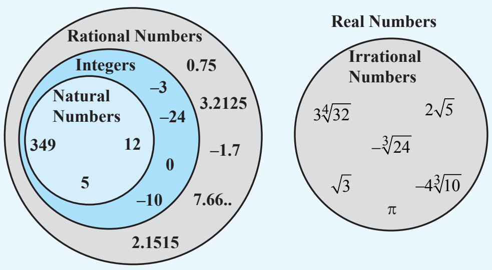
Chapter 1

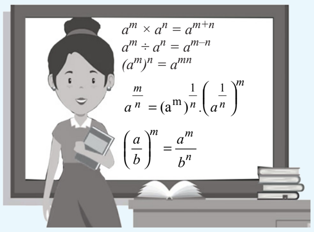
Number Systems

Learning Objectives

- Rational Numbers
  - *Finding rational numbers between 2 rational numbers.*
- Irrational Numbers
  - *Representation of Irrational numbers on the number line.*
- Real Numbers
  - *Conversion of rational numbers of the form $\frac{p}{q}$ to decimal form.*
  - *Conversion of rational number in decimal form to its simplest $\frac{p}{q}$ form*
- Properties of irrational numbers
  - *Rationalising factor*
- Laws of Rational exponents
  - *Conversion of rational number in decimal form to its simplest $\frac{p}{q}$ form*







Exam Mirror

- Finding rational numbers between 2 rational numbers.
- Conversion of rational number in decimal form to simplest $\frac{p}{q}$ form.
- Reducing a given surd to Rational form by rationalising.
- Representation of irrational number on real number line.
- Application of laws of rational exponents.

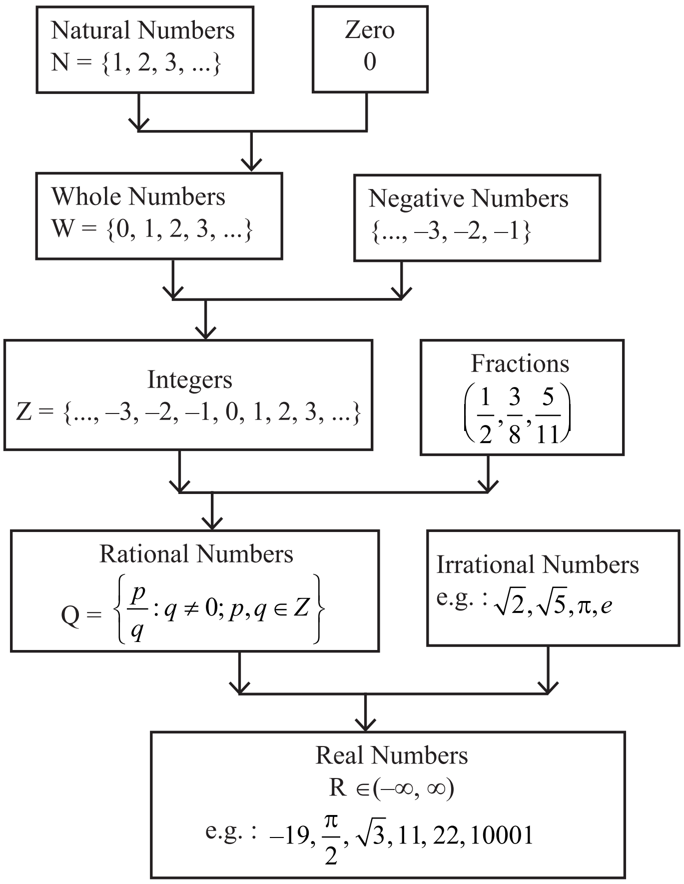
Critical Concepts

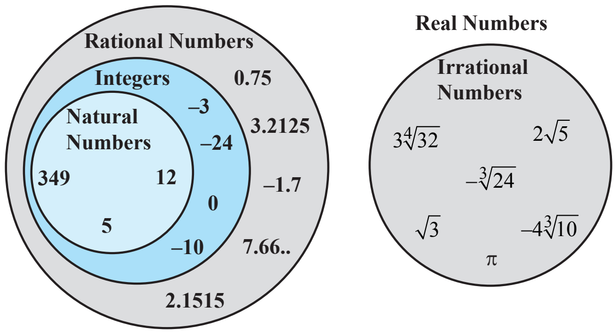
- Finding rational numbers between two rational numbers.
- Conversion of rational number in decimal form to simplest $\frac{p}{q}$ form
- Representation of irrational number on number line.
- Rationalising factor

INTRODUCTION

The collection of all types of numbers is called the number system. All numbers starting from 1 till infinity are natural numbers such as 1, 2, 3, ... ∞. All natural number including 0 are called whole number. Integers are whole numbers and negative whole numbers. The numbers which can be expressed as $\frac{p}{q}$ where p and q are integers and q ≠ 0 are called rational numbers. The numbers which cannot the expressed as $\frac{p}{q}$ when p and q are integer p ≠ 0 are called irrational numbers. Real numbers are collection of rational and irrational numbers.

NUMBER SYSTEMS



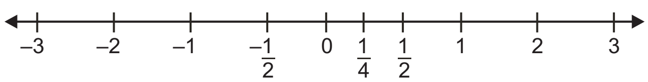


DID YOU KNOW?

A perfect number is a positive integer that is equal to the sum of its positive divisors. By this rule, 6 is the smallest perfect number.

NUMBER LINE

Representation of various types of numbers on the number line.



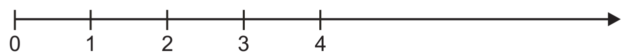
Various Types of Numbers

1. Set of natural numbers, N = {1, 2, 3, …}

representation of N on number line

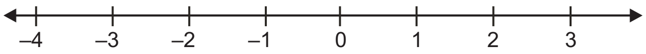
2. Set of whole numbers, W = {0, 1, 2, 3, …}

number line of W (whole numbers) :



3. Set of integers,
Z = {…, –3, –2, –1, 0, 1, 2, 3, … }

number line of Z (integers) :



4. **Rational numbers :** A number 'Q' is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Rational numbers, $Q = \left\{\frac{p}{q} : p, q \in I, q \neq 0\right\}$

> **DID YOU KNOW?**
> *The rational numbers also include the natural numbers, whole numbers and integers.*

FINDING RATIONAL NUMBERS BETWEEN TWO NUMBERS

Method to find two or more rational numbers between two numbers *p* and *q*:
In general, there are infinitely many rational numbers between any two given rational numbers.

If $p < q$, then one of the number be $p < \frac{p+q}{2} < q$ and others will be in continuation as $p < \frac{p+(p+q)/2}{2} < \frac{p+q}{2}$

Illustration 1 :

Find six rational numbers between $\frac{4}{7}$ and $\frac{1}{7}$.

Solution :

Denominator of both the given rational numbers $\frac{4}{7}$ and $\frac{1}{7}$ are equal. Now the difference of numerator = 4 – 1 = 3, which is not greater than 6 (i.e. the number of rational numbers is to be found out). On multiplying both numerator and denominator of $\frac{4}{7}$ and $\frac{1}{7}$ by 3, we get

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}, \quad \frac{1}{7} = \frac{1 \times 3}{7 \times 3} = \frac{3}{21}$$

Now the difference of numerator of $\frac{12}{21}$ and $\frac{3}{21}$ = 12 – 3 = 9, which is greater than 6.

Also, $\frac{3}{21} < \frac{12}{21}$ therefore $\frac{4}{21}, \frac{5}{21}, \frac{6}{21}, \frac{7}{21}, \frac{8}{21}$ and $\frac{9}{21}$ are one set of six rational numbers between $\frac{4}{7}$ and $\frac{1}{7}$.

Illustration 2 :

Find 3 rational numbers between 1 and 2.

Solution :

$1 = \frac{3}{3}$; $2 = \frac{6}{3}$

3 rational number between 1 and 2 are $\frac{2}{3}, \frac{5}{3}$ and $\frac{1+2}{2} = \frac{3}{2}$

CHECK POINT-1

1. Which of the following statement is not true?

(a) Between two integers, there exist infinite number of rational numbers

(b) Between two rational numbers, there exist infinite number of integers

(c) Between two rational numbers, there exist infinite number of rational numbers

(d) Between two real numbers, there exists infinite number of real numbers

2. Which of the following is correct ?

(a) Every whole number is a natural number.

(b) Every integer is a rational number.

(c) Every rational number is an integer.

(d) Every rational number is a whole number.

3. Four rational numbers between 3 and 4 are:

(a) $\frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}$ (b) $\frac{13}{5}, \frac{14}{5}, \frac{16}{5}, \frac{17}{5}$ (c) 3.1, 3.2, 4.1, 4.2 (d) 3.1, 3.2, 3.8, 3.9

Solutions:

1. (b) Between two rational number there exists infinite number of integers is not true.

2. (b) Every integer is a rational number

3. (d)

IRRATIONAL NUMBERS

A number is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

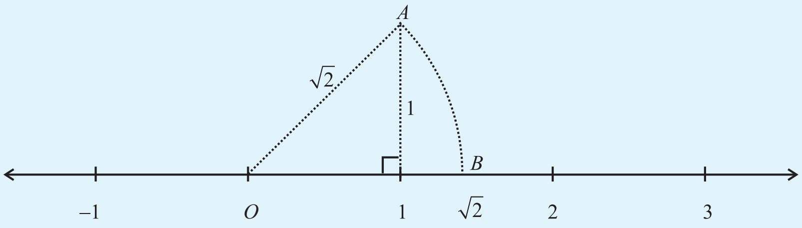
Examples are : $\sqrt{2}, \sqrt{3}, \sqrt{15}$, π, 0.10110111011110...

DID YOU KNOW?

When we use the symbol $\sqrt{\ }$, we assume that it is the positive square root of the number. So $\sqrt{4}$ = 2, though both 2 and – 2 are square roots of 4. i.e. $\sqrt{x}$ will always be a positive value.

Illustration 3 :

Represent of $\sqrt{2}$ on the Number Line



Solution :

Draw a number line indicating all integral points on it.

Since $\sqrt{2} > 0$ therefore, $\sqrt{2}$ lies right side of O on the number line.

Now, $\sqrt{2} = \sqrt{(1)^2 + (1)^2}$

So, if we draw a right angled triangle whose base and perpendicular each is of unit length, then length of its hypotenuse is $\sqrt{2}$ units. Draw a perpendicular line segment of unit length above the number line at the point representing 1 on it. Let the top-point of this perpendicular line segment be A. Clearly, $OA = \sqrt{2}$ units.

Now draw an arc with O as centre and OA as radius intersecting the number line at a point B on the right side of 1 on the number line. Hence, $OB = \sqrt{2}$ unit.

Therefore point B on the number line represent $\sqrt{2}$

Illustration 4 :

Represent of $\sqrt{3}$ on the Number Line

Solution :

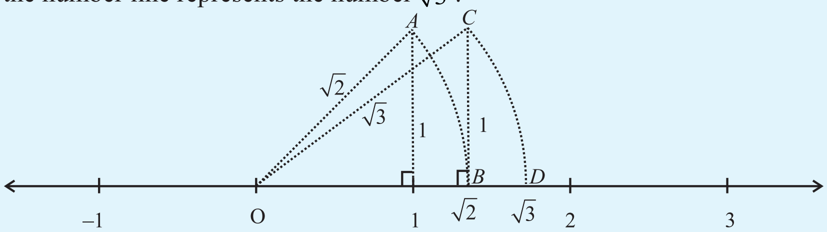
Since $\sqrt{3} = \sqrt{(\sqrt{2})^2 + 1^2}$

Mark a point B on the number line which represent $\sqrt{2}$ on the number line as discussed above.

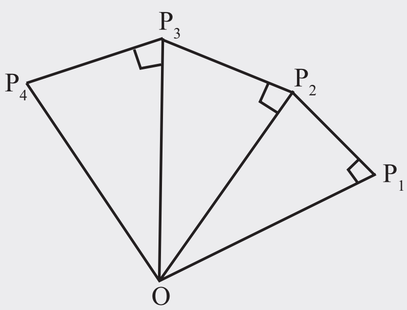
Draw a perpendicular line segment of unit length above the number line at the point representing $\sqrt{2}$ on it.

Let top point of this perpendicular line segment be C. Clearly $OC = \sqrt{3}$ units.

Taking O as centre and OC as radius, draw an arc intersecting the number line at point D on the right side of $\sqrt{2}$ on the number line. Clearly point D on the number line represents the number $\sqrt{3}$.



Let's Do Activity



- *Take a large sheet of paper and construct the 'square root spiral' in the following fashion.*
- *Start with a point O and draw a line segment OP, of unit length.*
- *Draw a line segment P_1, P_2 perpendicular to OP_1 of unit length.*
- *Now draw a line segment P_2P_3 perpendicular to OP_2.*
- *Continuing is this manner, you can get the line fegment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}.*
- *In this manner, you will have created the points P_2, P_3, ... P_n and join them to create a beautiful spiral depicting $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$.*

CHECK POINT-2

1. Which of the following is not correct ?

(a) There are infinitely many rational numbers between any two given rational numbers.
(b) Every point on the number line represents a unique real number.
(c) The decimal expansion of an irrational number is non-terminating non-recurring.
(d) A number whose decimal expansion is non-terminating non-recurring is rational.

2. The number 1.101001000100001....is

(a) a natural number (b) a whole number
(c) a rational number (d) an irrational number

3. $\left(-5+2\sqrt{5}-\sqrt{5}\right)$ is

(a) an irrational number (b) a positive rational number
(c) a negative rational number (d) an integer

4. The decimal expansion of the number $\sqrt{2}$ is

(a) a finite decimal (b) 1.41421
(c) non-terminating recurring (d) non-terminating non-recurring

Solutions:
1. (d) **2.** (d) **3.** (a) 4. (d)

CONNECTING TOPIC

To prove $\sqrt{n}$ is not a rational number, if n is not a perfect square

Let n be a whole number but not a perfect square

If possible, let $\sqrt{n}$ be the rational number, then it can be represented in the form of $\frac{p}{q}$, where p and q are integers but $q \neq 0$.

Also suppose $\frac{p}{q}$ is in simplest form, hence p and q are co-prime i.e. they have no common factor other than 1.

Now, $\frac{p}{q} = \sqrt{n}$

Squaring both the sides, we get $\frac{p^2}{q^2} = n \Rightarrow p^2 = nq^2$ ...(i)

Since n is a factor of p^2, $\therefore$ n is a factor of p.

Hence, we can suppose that $p = nm$, where m is an integer.

Putting the value of p in equation (i), we get

$n^2m^2 = nq^2 \Rightarrow q^2 = nm^2$

$\therefore$ n is a factor of q^2

$\Rightarrow$ n is a factor of q. ...(ii)

Hence, n is a factor of both p and q. This contradicts our assumption that p and q does not have any common factor.

This means our assumption that $\sqrt{n}$ is a rational number is wrong. Hence $\sqrt{n}$ is an irrational number, where n is not a perfect square.

To prove $\sqrt{2}$ is an irrational number

This is proved by the method of contradiction.

Let us suppose, if possible that $\sqrt{2}$ is not an irrational number i.e. $\sqrt{2}$ is a rational number. Hence, it can be represented in the form of $\frac{p}{q}$ i.e. $\sqrt{2} = \frac{p}{q}$; where p and q are integers but $q \neq 0$.

Let $\frac{p}{q}$ is in simplest form hence p and q are co-primes i.e. they have no common factor other than 1.

Squaring both the sides, we get $2 = \frac{p^2}{q^2}$

$\Rightarrow \quad p^2 = 2q^2$ ...(i)

Since RHS of (i) is twice the square of an integer, therefore RHS of (i) is even

$\Rightarrow \quad p^2$ is even

Hence p is also an even integer.

Let $p = 2m$, where m is an integer.

$p^2 = 4\,m^2$

Putting the value of p^2 in (i),

$4\,m^2 = 2\,q^2$

$\Rightarrow \quad q^2 = 2m^2$ ...(ii)

Since RHS of (ii) is an even integer, hence L.H.S. of it i.e. q^2 is also even integer.

Therefore, q is also an even integer.

Hence, p and q both are even integers.

But two even integers have always a common factor 2, which contradicts our assumption that p and q are co-primes.

Hence, $\sqrt{2}$ is an irrational number.

To prove $(2+\sqrt{3})$ is an irrational number

Let us suppose if possible that $2+\sqrt{3}$ is not an irrational number i.e. $2+\sqrt{3}$ is a rational number, then it can be represented in the form of $\frac{p}{q}$ where p and q are integers but $q \neq 0$.

i.e. $2+\sqrt{3} = \frac{p}{q} \Rightarrow \sqrt{3} = \frac{p}{q} - 2 = \frac{p-2q}{q}$ ...(i)

Since p and q are integer so $(p-2q)$ will also be, an integer, hence $\frac{p-2q}{q}$ is a rational number.

But $\sqrt{3}$ is an irrational.

This contradicts our assumption that $(2+\sqrt{3})$ is a rational number.

Hence, $(2+\sqrt{3})$ is an irrational number.

Let's Connect

1. Show that $5\sqrt{3}$ is an irrational number.

2. Prove that $\sqrt{3}+\sqrt{2}$ is irrational.

Solutions:

1. If possible, let $5\sqrt{3}$ be a rational number.

So, $5\sqrt{3} = p/q$ where p and q are co-prime integers and $q \neq 0$

So, $\sqrt{3} = \frac{p}{5q}$

So, R.H.S is a rational number and hence $\sqrt{3}$ is also rational which is a contradiction.

So our supposition is wrong. Hence, $5\sqrt{3}$ is an irrational number.

2. Let $\sqrt{3}+\sqrt{2} = r$ be a rational number.

$\Rightarrow \quad (\sqrt{3}+\sqrt{2})^2 = r^2$

$\Rightarrow \quad 3+2+2\sqrt{6} = r^2 \Rightarrow 2\sqrt{6} = r^2 - 5 \Rightarrow \sqrt{6} = \frac{r^2-5}{2}$

As R.H.S is rational, $\sqrt{6}$ should be rational which is incorrect.

Hence, $\sqrt{3}+\sqrt{2}$ is an irrational number.

REAL NUMBERS

The set of rational numbers and irrational numbers form a set of real numbers which is denoted by R.

Remark

Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

DID YOU KNOW?

(i) All integers are rational numbers.

(ii) The square root of every perfect square number is rational. e.g. $\sqrt{4}=2, \sqrt{9}=3, \sqrt{16}=4$ etc. are all rational numbers.

(iii) The square root of any positive number which is not a perfect square is an irrational number.

E.g.: $\sqrt{5}, \sqrt{3}, \sqrt{10}, \sqrt{12}, \sqrt{3.4}, \sqrt{0.748}$ etc.

(iv) π is an irrational number, which is actually the ratio of circumference to the diameter of a circle i.e. $\pi=\frac{c}{d}$, where c and d are the circumference and diameter of a circle. Approximate value of π is taken as $\frac{22}{7}$ or 3.14

DECIMAL REPRESENTATION OF RATIONAL NUMBERS IN THE FORM $\frac{p}{q}$

Every rational number in the form $\frac{p}{q}$ can be expressed to its equivalent decimal form.

For example, $\frac{1}{5}=0.2, \quad \frac{1}{3}=0.333.....=0.\overline{3}$, etc

Generally, we use long division method to get the decimal form.

For example to get the decimal form of $\frac{13}{7}$, we simply divide 13 by 7 is shown below

$$
\begin{array}{r|l}
7 & 13 \;(\; 1.857142 \\
 & \underline{7} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{35} \\
 & 50 \\
 & \underline{49} \\
 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 6
\end{array}
$$

$\therefore \quad \frac{13}{7}=1.857142857142.... \;=1.\overline{857142}$

Terminating Decimal Expansions

In this case, the decimal expansion terminates or ends after a finite number of steps. We call such a decimal expansion as terminating.

Non-terminating Recurring Expansions

In this case we have a repeating block of digits in the quotient. We say that this expansion is non-terminating recurring.

DID YOU KNOW?

The decimal number system which we study today was introduced to the Islamic civilization by Al-Khwarizmi. It was known as Hindu numeral system. It was only centuries later, in the 12th century, that the Indian numeral system was introduced to the Western world by the Arabs, and it was named as Hindu-Arabic numeral system.

CONVERSION OF RATIONAL NUMBER IN DECIMAL FORM TO ITS SIMPLEST $\frac{p}{q}$ FORM

Type-I : Conversion of a Terminating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Obtain the rational number.

Step 2 : Determine the number of digits in its decimal part.

Step 3 : Remove decimal point from the given number and write 1 as its denominator followed by as many zeros as the total number of digits in the decimal part of the given number.

Step 4 : Write the number obtained in step-3 in its simplest form (i.e. the form in which there is no common factor other than 1 in its numerator and denominator).

The number so obtained is the required $\frac{p}{q}$ form.

Illustration 5 :

Convert rational number 2.348 in simplest $\frac{p}{q}$ form.

Solution :

Given rational number = 2.348. There are three digits in the decimal part.

$\therefore 2.348 = \frac{2348}{1000}$. Now, write $\frac{2348}{1000}$ in its lowest form.

$2.348 = \frac{2348}{1000} = \frac{1174}{500} = \frac{587}{250}$, which is the required form.

Type-II : Conversion of Non-terminating Repeating Decimal Number to its Simplest $\frac{p}{q}$ Form.

Step 1 : Put the given number equal to x and consider it as equation number say (i)

Step 2 : If there are some non-repeating digits after decimal on the right hand side of the equation (i), then count the number of non-repeating digits after the decimal point. Let it be n. Otherwise go to step 4 directly.

Step 3 : Multiply both sides of equation (i) by $(10)^n$, so that on the right hand side of the decimal point only the repeating digit(s) is/are left. Consider the equation so obtained as equation number (ii).

Step 4 : Multiply both sides of the equation (ii) by $(10)^m$, where m is the number of digit(s) which repeats on the right hand side after decimal point. Consider the equation so obtained as equation number (iii).

Step 5 : Subtract the equation (ii) from equation (iii).

Step 6 : Divide both sides of the equation obtained in step-5 by the coefficient of x.

Step 7 : Write the rational number on the right hand side of equation obtained in step-6 in the simplest $\frac{p}{q}$ form. This simplest $\frac{p}{q}$ form is the required form.

Illustration 6 :

Express 6.48261261261………. in the simplest $\frac{p}{q}$ form.

Solution :

Let $x = 6.48261261261……$

$\Rightarrow \; x = 6.48\overline{261}$ ...(i)

Since there are two non-repeating digits (48) on the right hand side of equation (i), therefore we multiply both sides of equation (i) by $(10)^2$ i.e. 100, we get

$100\,x = 648.\overline{261}$ …(ii)

Since there are three repeating digits (261), therefore multiply both sides of equation (ii) by $(10)^3$ i.e. 1000, we get

$100000x = 648261.\overline{261}$ … (iii)

Subtracting equation (ii) from (iii), we get $99900x = 648261 - 648$

$\Rightarrow \quad 99900x = 647613$ ...(iv)

Divide both side of equation (iv) by the coefficient 99900 of x in equation (iv),
we get

$$\frac{99900x}{99900} = \frac{647613}{99900} \Rightarrow x = \frac{647613}{99900} \Rightarrow x = \frac{71957}{11100} \text{ (in simplest form)}$$

Hence, $6.48261261261……………… = \frac{71957}{11100}$, which is the required $\frac{p}{q}$ form.

Illustration 7 :

Express $0.\overline{52}$ in the $\frac{p}{q}$ form.

Solution :

Let $\quad x = 0.\overline{52}$

$x = 0.525252$ ...(i)

There is no non-repeating digit after decimal point on the right hand side in equation (i).

Number of repeating digits after the decimal point on the right hand side of equation (i) is 2. Hence, multiplying both sides of equation (i) by $(10)^2$ i.e. 100, we get

$100\,x = 52.525252.....$ ...(ii)

Subtract (i) from (ii), we get

$$\begin{array}{r} 100x = 52.5252..... \\ x = 0.525252... \\ \hline 99x = 52 \end{array} \Rightarrow x = \frac{52}{99} \quad \therefore \; 0.\overline{52} = \frac{52}{99}$$

Illustration 8 :

Express 0.2434343……… in the form of $\frac{p}{q}$.

Solution :

Let $x = 0.2434343$ ……

$x = 0.2\overline{43}$ ...(i)

Here, a digit 2 does not repeat after decimal point.

So, we multiply equation (i) by 10, we get

$10x = 2.4343$ ...(ii)

Since there are two repeating digits (43) after decimal point in the right side of equation (ii).

So, multiplying (ii) by $(10)^2$ i.e. 100, we get

$1000x = 243.4343$ ...(iii)

Subtract (ii) from (iii), we get

$$\begin{aligned} 1000x &= 243.4343\ldots\ldots \\ 10x &= 2.4343\ldots\ldots \\ \hline 990x &= 241 \end{aligned}$$

$x = \frac{241}{990} \Rightarrow 0.2434343.......... = \frac{241}{990}$ is the required $\frac{p}{q}$ form.

CHECK POINT-3

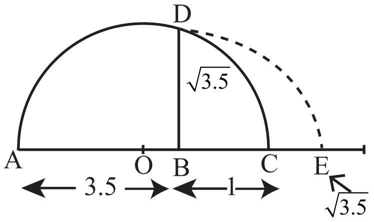
1. Convert $\frac{4}{11}$ into decimal form.
   (a) $0.\overline{36}$ (b) $0.37\overline{36}$ (c) $36.\overline{36}$ (d) $18.\overline{36}$
2. Irrational number between 1.011243... and 1.012243... is
   (a) 1.011143 ... (b) 1.012343 ... (c) 1.01152243 ... (d) 1.013243
3. Decimal representation of a rational number cannot be
   (a) terminating (b) non-terminating
   (c) non-terminating repeating (d) non-terminating non-repeating
4. Express the repeating decimal as a quotient of integer $0.1\overline{34}$.
   (a) $\frac{133}{990}$ (b) $\frac{133}{900}$ (c) $\frac{133}{999}$ (d) $\frac{143}{990}$

Solutions:

1. (a) 2. (c) 3. (d) 4. (a)

How to represent square root of any rational number on number line.

Eg. To find $\sqrt{3.5}$



Steps

1. Make the distance 3.5 units from a fixed point A on a given line to obtain a point B such that B = 3.5 units.
2. From B, mark a distance of 1 unit and mark the new point as C.
3. Find mid point of AC and mark that point as O.
4. Draw a semicircle with centre O and radius OC.
5. Draw a line perpendicular to AC passing through B and intersecting semicircle at D. Then BD = 3.5
6. Now let us treat BC as the number line, with B as zero, and C as 1 and so on. Draw an arc with centre B and radius BD which intersect the number line in E. Then E represents $\sqrt{3.5}$.

PROPERTIES OF IRRATIONAL NUMBERS

(i) Negative of an irrational number is also an irrational number.

For example: $-\sqrt{5}$ is an irrational number, because $\sqrt{5}$ is an irrational number.

(ii) The sum or difference of a rational number and an irrational number is an irrational.

For example : $2+\sqrt{3}$, $2-\sqrt{3}$, $\sqrt{3}-2$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iii) The product or quotient of a non-zero rational number with an irrational number is an irrational.

For example : $2\sqrt{3}, \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{2}$ are irrational numbers because 2 is a rational number and $\sqrt{3}$ is an irrational number.

(iv) The sum, difference, product and quotient of two irrational numbers may be rational or irrational.

For examples :

(i) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers but their sum $= 2+\sqrt{3}+2-\sqrt{3} = 4$ is a rational number.

(ii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers but their sum $=(2+\sqrt{3})+(-2+\sqrt{3}) = 2\sqrt{3}$ is an irrational number.

(iii) $(2+\sqrt{3})$ and $(-2+\sqrt{3})$ are two irrational numbers but their difference $= (2+\sqrt{3})-(-2+\sqrt{3}) = 4$ is a rational number.

(iv) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two irrational numbers but their difference $= (2+\sqrt{3})-(2-\sqrt{3}) = 2\sqrt{3}$ is also an irrational number.

(v) $\sqrt{2}$ and $\sqrt{8}$ are two irrational numbers but their product $= \sqrt{2}\times\sqrt{8} = \sqrt{16} = 4$ or -4 is a rational number.

(vi) $\sqrt{2}$ and $\sqrt{3}$ are two irrational numbers but their product $= \sqrt{2}\times\sqrt{3} = \sqrt{6}$ is an irrational number.

(vii) $\sqrt{8}$ and $\sqrt{2}$ are irrational numbers but their quotient $=\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$ is a rational number.

(viii) $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are irrational numbers but their quotient $= \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})\times(2+\sqrt{3})}{(2-\sqrt{3})\times(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{4-3} = \frac{4+3+4\sqrt{3}}{1}$

$= 7+4\sqrt{3}$ is an irrational number.

DID YOU KNOW?

If a is the rational number and n is a positive integer such that the nth root of a is an irrational number, then a1/n is called a surd eg. $\sqrt{5}, \sqrt{2}$ *etc. If* $\sqrt[n]{a}$ *is a surd then 'n' is known as order of surd and 'a' is known as radicand. Every surd is an irrational number but every irrational number, is not a surd.*

SOME IDENTITIES RELATED TO SQUARE ROOTS

Let a and b be positive real numbers. Then

(i) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

(iv) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

(v) $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

(vi) $(\sqrt{a}+\sqrt{b})^2 = a+2\sqrt{ab}+b.$

RATIONALISING FACTOR

Rationalisation

When the denominator of an expression contains a term with a square root, the procedure of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

For example :

(i) $(\sqrt{a}+\sqrt{b})$ is the rationalising factor of $(\sqrt{a}-\sqrt{b})$ and vice-versa.

(ii) $(a+\sqrt{b})$ is the rationalising factor of $(a-\sqrt{b})$ and vice-versa.

Key concept here is : Using identity $(a + b)$ (a – $b) = a^2 - b^2$

Illustration 9 :

Rationalise the denominator of $\dfrac{1}{7+3\sqrt{2}}$

Solution :

We have,

$$\frac{1}{7+3\sqrt{2}}=\frac{1}{7+3\sqrt{2}}\times\frac{7-3\sqrt{2}}{7-3\sqrt{2}}=\frac{7-3\sqrt{2}}{(7)^2-(3\sqrt{2})^2}=\frac{7-3\sqrt{2}}{49-18}=\frac{7-3\sqrt{2}}{31}$$

Illustration 10 :

Simplify each of the following by rationalising the denominator $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Solution :

Multiplying both numerator and denominator by the rationalisation factor of the denominator, we have

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}=\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$=\frac{2\sqrt{3}\times2\sqrt{2}-2\sqrt{3}\times3\sqrt{3}-\sqrt{5}\times2\sqrt{2}+\sqrt{5}\times3\sqrt{3}}{(2\sqrt{2})^2-(3\sqrt{3})^2}=\frac{4\sqrt{3\times2}-6\sqrt{3\times3}-2\sqrt{5\times2}+3\sqrt{5\times3}}{4\times2-9\times3}$$

$$=\frac{4\sqrt{6}-6\times3-2\sqrt{10}+3\sqrt{15}}{8-27}=\frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}=\frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

CHECK POINT-4

1. Find the sum of the squares of the following: $\dfrac{\sqrt{3}}{\sqrt{2}+1},\dfrac{\sqrt{3}}{\sqrt{2}-1},\dfrac{\sqrt{2}}{\sqrt{3}}$

(a) $\dfrac{56}{4}$ (b) $18\dfrac{2}{3}$ (c) $2\dfrac{18}{3}$ (d) $3\dfrac{18}{2}$

2. The sum of rational and irrational number is:

(a) Rational (b) Irrational

(c) Zero (d) Integers

3. If $x=\dfrac{1}{1+\sqrt{2}}$, then the value of x^2+2x+3 is

(a) 3 (b) 0 (c) 4 (d) 1

4. If a and b are two rational numbers and $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}=a+b\sqrt{3}$, then what is the value of a – b?

(a) $\sqrt{3}$ (b) 4 (c) 7 (d) 3

Solutions:

1. (b) **2.** (b) **3.** (c) 4. (d)

LAWS OF RATIONAL EXPONENTS

If a & b are positive real numbers and m & n are rational numbers, then

(i) $a^m\times a^n=a^{m+n}$

(ii) $a^m\div a^n=a^{m-n}$

(iii) $(a^m)^n=a^{mn}$

(iv) $a^{-n}=\dfrac{1}{a^n}$

(v) $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$ i.e. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(vi) $(ab)^m = a^m b^m$

(vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(viii) $a^{m^n} \neq \left(a^m\right)^n$

Illustration 11 :

Simplify : $13^{\frac{1}{5}}.17^{\frac{1}{5}}$

Solution :

$13^{\frac{1}{5}}.17^{\frac{1}{5}} = (13 \times 17)^{\frac{1}{5}} = 221^{\frac{1}{5}} = \sqrt[5]{221}$

Illustration 12 :

Simplify: $\left(\frac{2^a}{2^b}\right)^{a+b}\left(\frac{2^b}{2^c}\right)^{b+c}\left(\frac{2^c}{2^a}\right)^{c+a}$

Solution :

$$\left(\frac{2^a}{2^b}\right)^{a+b}\left(\frac{2^b}{2^c}\right)^{b+c}\left(\frac{2^c}{2^a}\right)^{c+a} = (2^{a-b})^{a+b} \cdot (2^{b-c})^{b+c} \cdot (2^{c-a})^{c+a}$$

$$= 2^{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)} = 2^0 = 1$$

Illustration 13 :

Prove that $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$

Solution :

We have,

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = \frac{x^{ab-ac}}{x^{ba-bc}} \div (x^{bc-ac})$$

$$= x^{ab-ac-ba+bc} \times \frac{1}{x^{bc-ac}} = x^{ab-ba-ac+ac+bc-bc} = x^0 = 1$$

Illustration 14 :

Simplify : $\left(\frac{a^m}{a^n}\right)^{m+n}\left(\frac{a^n}{a}\right)^{n+1}\left(\frac{a}{a^m}\right)^{1+m}$

Solution :

$$\left(\frac{a^m}{a^n}\right)^{m+n}\left(\frac{a^n}{a}\right)^{n+1}\left(\frac{a}{a^m}\right)^{1+m}$$

$= (a^{m-n})^{m+n}\,(a^{n-1})^{n+1}\,(a^{1-m})^{1+m}$ $\quad [\because x^a \div x^b = x^{a-b}]$

$= a^{(m-n)(m+n)}\,.\,a^{(n+1)(n-1)}\,.\,a^{(1-m)(1+m)}$

$= a^{m^2-n^2}.\ a^{n^2-1}.\ a^{1-m^2}$ $\quad [\because (a+b)(a-b) = a^2 - b^2]$

$= a^{(m^2-n^2)+(n^2-1)+(1-m^2)}$ $\quad [\because (x^a \times x^b) = x^{a+b}]$

$= a^0 = 1$ $\quad [\because x^0 = 1]$

$$= \frac{4\sqrt{6}-6\times 3-2\sqrt{10}+3\sqrt{15}}{8-27} = \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

CHECK POINT-5

1. $\left(\frac{2}{3}\right)^{3/4}$ when divided by $\left(\frac{2}{3}\right)^{7/6}$ gives $\left(\frac{2}{3}\right)^{7-x}$. Find the value of x.

(a) $\frac{7}{12}$ (b) $\frac{89}{12}$ (c) $\frac{8}{12}$ (d) $\frac{10}{12}$

2. Find 'x', if $8^{x-2} \times \left(\frac{1}{2}\right)^{4-3x} = (0.0625)^x$.

(a) 0 (b) 4 (c) 2 (d) 1

3. $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} - (-32)^{4/5}$ is equal to

(a) $-15\frac{7}{8}$ (b) $16\frac{1}{8}$ (c) $-14\frac{7}{8}$ (d) $17\frac{1}{8}$

4. The value of x, when $(2)^{x+4}.(3)^{x+1} = 288$ is

(a) 1 (b) –1 (c) 0 (d) None

Solutions:

1. (b) **2.** (d) **3.** (c) 4. (a)

CASE STUDY : Understand Terminating or Non-terminating Rational numbers

CASE-I : Remainder becomes zero.

Rational numbers which have their denominator of the form 2^n or 5^m or $2^n.5^m$ have terminating decimals.

Let us understand the above concepts in a better way in different cases through illustrative examples.

Illustration: *Out of* $\frac{7}{8}, \frac{2}{5}, \frac{33}{160}, \frac{5}{250}$ *which has/have terminating decimals?*

Sol. In order to check terminating decimals, denominator must be of the form 2^n or 5^m or $2^n.5^m$, let us check each denominator one by one

$8 = 2^3$, thus the first term will have terminating decimal.

$\frac{7}{8} = 0.875$

$5 = 5^1$, thus the second term will have terminating decimal.

$\frac{2}{5} = 0.4$

$160 = 2^5.5^1$, thus the third term will have terminating decimal.

$\frac{33}{160} = 0.01875$

$250 = 2^1.5^3$, thus the fourth term will have terminating decimal.

$\frac{5}{250} = 0.02$

CASE-II : Remainder is non-zero.

Rational numbers whose denominator is not of the form 2^n or 5^m or $2^n.5^m$ have non-terminating decimals.

Let us understand the above concepts in a better way in different cases through illustrative examples.

Illustration: *Out of* $\frac{4}{28}, \frac{2}{3}, \frac{3}{121}, \frac{5}{12}$ *which has/have non-terminating decimals?*

Sol. In order to check for non-terminating decimals, denominator must be of the form other than 2^n or 5^m or $2^n.5^m$, let us check each denominator one by one -

$28 = 2^2 \times 7$, thus the first term will have non-terminating decimal.

$\frac{4}{28} = 0.142857....$

$3 = 3^1$, thus the second term will have non-terminating decimal.

$\frac{2}{3} = 0.66666666...$

$121 = 11^2$, thus the third term will have non-terminating decimal.

$\frac{3}{121} = 0.024793388...$

$12 = 2^2.3$, thus the fourth term will have non-terminating decimal.

$\frac{5}{12} = 0.416666...$

Think Out of the Box

Q.1 Which of the following rational numbers can be represented as terminating decimals?

(a) 3/5 (b) 2/13 (c) 7/20 (d) none of these

[**Hint :** Check if the denominator can be expressed in the form 2^n or 5^m or $2^n. 5^m$. **Ans.** (a, c)]

Q.2 The number of consecutive zeros in $2^3 \times 3^4 \times 5^4 \times 7$ is

(a) 3 (b) 2 (c) 4 (d) 5

[**Hint :** Count the number of factor 2×5 (i.e., 10). **Ans.** (3)]

Walk Through the Chapter

Number System

Real Numbers

The set of rational numbers and irrational numbers from a set of real numbers which is denoted by R. Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

Rational Numbers

Numbers which can be expressed in the form of $\frac{p}{q}$ where p, q are integers and q ≠ 0

Irrational Numbers

A number 's' is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Examples are: $\sqrt{2}, \sqrt{3}, \sqrt{15}$, π, 0, 10110111011110...

Representation of Irrational Numbers on the Number Line

To represent an irrational number in the form $\frac{p}{q}$, we use the Pythagoras theorem of a right angle triangle, according to which in a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides.

i.e. $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{perpendicular})^2$

Rationalisation

The process of reducing a given surd to a rational form after multiplying it by a suitable surd is known as rationalisation. When the product of two surds is a rational number, then each of the two surds is called rationalising factor of the other.

Laws of Rational Exponents

If a & b are positive real numbers and m & n are rational numbers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^{-n} = \frac{1}{a^n}$

(v) $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$

(vi) $(ab)^m = a^m b^{mn}$

(vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Let's Revise Through FIB & T/F

1. 0.578 is ______ number. (rational/irrational)
2. Between two rational numbers, there exist __________ number of rational numbers.
3. Between two real numbers, there exists infinite number of __________ numbers.
4. Every whole number is a natural number. **(T/F)**
5. Every rational number is an integer. **(T/F)**
6. Every natural number is a whole number. **(T/F)**
7. Every integer is a whole number. **(T/F)**
8. Every rational number is a whole number. **(T/F)**
9. All rational numbers can be represented by some point on the number line. **(T/F)**
10. Value of a is __________ if $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$
11. Two mixed quadratic surds, $a+\sqrt{b}$ and $a-\sqrt{b}$, whose sum and product are rational, are called _________.
12. If $x+\sqrt{5} = 4+\sqrt{y}$, then $x+y =$ ______. (where x and y are rational)
13. The sum/difference of a rational and an irrational number is ______.
14. All rational numbers when expressed in decimal form are either terminating decimals or repeating decimals. **(T/F)**
15. The sum or difference of a rational number and an irrational number is an irrational. **(T/F)**
16. A real number is either rational or irrational. **(T/F)**
17. Product of a rational and an irrational number is always irrational. **(T/F)**
18. An irrational number between $\frac{2}{5}$ and $\frac{3}{7}$ is
19. $\sqrt[4]{\frac{1008}{63}}$ is equal to __________
20. 0.72737475...... is ______ number. (rational/irrational)

EXERCISE -1 Master Board

Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct.*

1. $0.12\overline{3}$ can be expressed in rational form as

(a) $\frac{900}{111}$ (b) $\frac{111}{900}$ (c) $\frac{123}{10}$ (d) $\frac{121}{900}$

2. Find x^2, if $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

(a) $\frac{3}{2}$ (b) 1 (c) 2 (d) 4

3. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$ then find the value of m.

(a) $-\frac{1}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{1}{7}$ (d) $\frac{2}{8}$

4. Let x and y be rational an irrational number respectively, then x + y is necessarily

(a) a whole number

(b) a rational number

(c) an irrational number

(d) a natural number.

5. $0.12\overline{3}$ can be expressed in rational form as

(a) $\frac{900}{111}$ (b) $\frac{111}{900}$ (c) $\frac{123}{10}$ (d) $\frac{121}{900}$

6. If a and b are two positive rational number then $\sqrt{\frac{a+b}{2}}$ is .... number.

(a) Rational (b) Irrational

(c) Integer (d) Rational or irrational

7. Two rational numbers between 6 and 9.

(a) $\frac{14}{2}, \frac{3}{2}$ (b) $\frac{21}{3}, \frac{24}{3}$ (c) $\frac{40}{5}, \frac{14}{2}$ (d) $\frac{7}{3}, \frac{8}{3}$

8. If $x = 2 + \sqrt{3}$, $xy = 1$, then find $\frac{x}{2-x} + \frac{y}{2-y}$

(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $-\sqrt{2}$ (d) -2

9. What is the value of

$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}}$.... upto 15 terms?

(a) 4 (b) 0 (c) 2 (d) 3

10. If $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$ and $\sqrt{6} = 2.449$, find the value of

$\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

(a) 14.268 (b) 18.428 (c) 14.629 (d) 14.662

11. If $x - \frac{1}{x} = \sqrt{3}$, then $x^3 - \frac{1}{x^3}$ equals

(a) $6\sqrt{3}$ (b) $3\sqrt{3}$ (c) 3 (d) $\sqrt{3}$

12. The rational form of $2.74\overline{35}$ is

(a) $\frac{27161}{999}$ (b) $\frac{27}{99}$ (c) $\frac{27161}{9900}$ (d) $\frac{27161}{9000}$

Assertion & Reason Questions

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(c) If **Assertion** is **correct** but **Reason** is **incorrect.**

(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{5}{12}$.

Reason : Rational number between two numbers a and b is $\sqrt{ab}$.

2. **Assertion:** $5 - \sqrt{2} = 5 - 1.414 = 3.586$ is irrational number.

Reason: The difference of a rational number and an irrational number is an irrational number.

3. **Assertion:** $\sqrt{3}$ is an irrational number.

Reason : The sum of a rational number and an irrational number is an irrational number.

Very Short Answer Questions

1. Write the smallest whole number.
2. Write the integer other than 1, which is a reciprocal of itself.
3. Suppose a is a rational number. What is the reciprocal of the reciprocal of a ?
4. Write the repeating decimal for each of the following, and use a bar to show the repetend.

(i) $-\frac{4}{3}$ (ii) $\frac{11}{12}$ (iii) $\frac{7}{13}$

5. Classify the following numbers as rational or irrational.

(i) $\sqrt{225}$ (ii) 7.478478......

6. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
7. Simplify :

(i) $\left(\frac{1}{3^3}\right)^7$ (ii) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

8. Simplify :

(i) $(\sqrt{4})^{3/4}$ (ii) $\left(\frac{5}{8}\right)^3\left(\frac{4}{3}\right)^3$ (iii) $\left[\frac{\frac{\sqrt{5}}{3}}{\frac{5}{9}}\right]^6$

(iv) $\left(\frac{5}{3}\right)^3.\left(\frac{9}{2}\right)^4$ (v) $\frac{2^6 \times 8^2}{4^4}$

9. Find two irrational numbers between 0.1 and 0.2.
10. Determine, without actually dividing, which of the following rational numbers can be named, (a) by a terminating decimal, (b) by a repeating decimal.

(i) $\frac{7}{20}$ (ii) $\frac{1}{6}$

(iii) $\frac{1}{12}$ (iv) $3\frac{47}{160}$

11. Write down a fraction which is equivalent to 0.033636363..........
12. Find two rational numbers between 0.222332333233332.... and 0.252552555255552....
13. Which is greatest : $\sqrt[3]{4}$, $\sqrt[4]{5}$ or $\sqrt[4]{3}$?
14. Find four rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$.

Short Answer Questions

1. If $\frac{3+\sqrt{5}}{4-2\sqrt{5}} = p+q\sqrt{5}$, where p and q are rational numbers, find the values of p and q.
2. Simplify:

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

3. Examine whether the following numbers are rational or irrational :

(i) $(2-\sqrt{3})^2$ (ii) $(3+\sqrt{2})(3-\sqrt{2})$

4. Find the value of 'a' in the following expression.

$$\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2}-a\sqrt{3}$$

5. Simplify : $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

6. Examine whether the number is rational or irrational $\frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})}$.

7. Given that $\sqrt{3} = 1.732$ find the value of

$$\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192}$$

8. Simplify and express the result in simplest form

$$\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \div \frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$$

9. Find x^2, if $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$
10. Show how $\sqrt{5}$ can be represented on the number line.
11. Show that $\frac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4} = 1$

12. If $x = 3\sqrt{3} + \sqrt{26}$, then find the value of $\frac{1}{2}\left(x + \frac{1}{x}\right)$

13. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$, then find the value of m.

14. Find two irrational numbers between 0.12 and 0.13.

15. Find three rational numbers between $\frac{1}{5}$ and $\frac{7}{10}$.

16. Express $1.272727..... = 1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

17. If $x = \sqrt{2} - \sqrt{3}$, then find the value of $x + \frac{1}{x}$.

18. If $6^x = 30$, then what will be the value of $6^{x-1} + 6^{x+1}$?

Long Answer Questions

1. Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

2. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$. Then find the value of $x^2 + y^2$.

3. Express with a rational denominator :

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

4. Simplify : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

5. Represent $\sqrt{9.3}$ on number line.

6. Find the value of $2.\overline{6} - 0.\overline{9}$

7. Rationalise the denominator and simplify :

$$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$$

8. If $x = \frac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 3$

9. Express $0.12\overline{3}$ in $\frac{p}{q}$ form.

10. If $9^{a+1} = 81^{b+2}$ and $\left(\frac{1}{3}\right)^{3+a} = \left(\frac{1}{27}\right)^{3b}$ and the values of a and b.

11. Prove that $\frac{16 \times 2^{x+1} - 4 \times 2^x}{16 \times 2^{x+2} - 2 \times 2^{x+2}} = \frac{1}{2}$

12. Simplify : $\left(\sqrt{324 + 2\sqrt{323}}\right) - \left(\sqrt{324 - 2\sqrt{323}}\right)$

HOTS Questions

1. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

2. Express 2.5434343… in the form p/q where p and q are integers and $q \neq 0$

3. If $x = \frac{1}{7+4\sqrt{3}}, y = \frac{1}{7-4\sqrt{3}}$, find the value of $5x^2 - 7xy - 5y^2$.

EXERCISE -2 NCERT Questions

Text Book Questions

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

2. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

3. State whether the following statements are true or false? Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

4. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

5. Are the square roots of all positive integers irrational ? If not, give an example of the square roots of a number that is a rational number.

6. Find : $125^{1/3}$

7. Find :

(i) $32^{2/5}$

(ii) $125^{-1/3}$

8. Simplify :

(i) $2^{\frac{2}{3}} . 2^{\frac{1}{5}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

(iii) $\frac{11^{1/2}}{11^{1/4}}$

Exemplar Questions

1. Every rational number is

(a) a natural number (b) an integer

(c) a real number (d) a whole number

2. Between two rational numbers

(a) there is no rational number

(b) there is exactly one rational number

(c) thre are infinitely many rational numbers

(d) there are only rational numbers and no irrational numbers.

3. Decimal representation of a rational number cannot be

(a) terminating

(b) non-terminating

(c) non-terminating repeating

(d) non-terminating non-repeating

4. The product of any two irrational number is

(a) always an irrational number

(b) always a rational number

(c) always an integer

(d) sometimes rational, sometimes irrational

5. The decimal expansion of the number is $\sqrt{2}$ is

(a) a finite decimal

(b) 1.41421

(c) non-terminating recurring

(d) non-terminating non-recurring

6. Which of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$

(b) $\sqrt{\frac{12}{3}}$

(d) $\sqrt{7}$

(d) $\sqrt{81}$

7. Which of the following is irrational?

(a) 0.14

(b) $0.1\overline{416}$

(d) $0.\overline{1416}$

(d) 0.4014001400014...

8. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(a) $\dfrac{\sqrt{2}+\sqrt{3}}{2}$

(b) $\dfrac{\sqrt{2}\times\sqrt{3}}{2}$

(c) 1.5

(d) 1.8

9. $2\sqrt{3}+\sqrt{3}$ is equal to

(a) $2\sqrt{6}$

(b) 6

(c) $3\sqrt{3}$

(d) $4\sqrt{6}$

10. $\dfrac{1}{\sqrt{9}-\sqrt{8}}$ is equal to

(a) $\dfrac{1}{2}(3-2\sqrt{2})$

(b) $\dfrac{1}{3+2\sqrt{2}}$

(c) $3-2\sqrt{2}$

(d) $3+2\sqrt{2}$

11. Simplify the following :

(i) $3\sqrt{3}+2\sqrt{27}+\dfrac{7}{\sqrt{3}}$

(ii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$

12. Find the value of:

$$\frac{4}{(216)^{\frac{-2}{3}}}+\frac{1}{(256)^{\frac{-3}{4}}}+\frac{2}{(243)^{\frac{-1}{5}}}$$

EXERCISE -3 Foundation Builder

Multiple Choice Questions

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

1. The value of x, when $(2)^{x+4}.(3)^{x+1} = 288$ is
(a) 1 (b) –1
(c) 0 (d) None

2. The value of 0.423 is
(a) $\frac{423}{1000}$ (b) $\frac{423}{100}$
(c) $\frac{423}{990}$ (d) $\frac{419}{990}$

3. If $a = 2+\sqrt{3}$ and $b = 2-\sqrt{3}$, then $\frac{1}{a^2}-\frac{1}{b^2}$ is equal to,
(a) 14 (b) – 14
(c) $8\sqrt{3}$ (d) $-8\sqrt{3}$

4. Value of x satisfying $\sqrt{x+3}+\sqrt{x-2} = 5$, is
(a) 6 (b) 7
(c) 8 (d) 9

5. A rational number equivalent to a rational number $\frac{7}{19}$ is
(a) $\frac{17}{119}$ (b) $\frac{14}{57}$
(c) $\frac{21}{38}$ (d) $\frac{21}{57}$

6. Rationalizing factor of $(2+\sqrt{3})$=
(a) $2-\sqrt{3}$ (b) $\sqrt{3}$
(c) $2+\sqrt{3}$ (d) $3+\sqrt{3}$

7. Rationalizing factor of $1+\sqrt{2}+\sqrt{3}$
(a) $1+\sqrt{2}-\sqrt{3}$ (b) 2
(c) 4 (d) $1+\sqrt{2}+\sqrt{3}$

8. Value of $\frac{2^{n+2}-2(2^n)}{2^{(2n+2)}}$ when simplified is
(a) $1-2(2^n)$ (b) $2^{n+3}-\frac{1}{4}$
(c) $\frac{1}{2^{n+1}}$ (d) $\frac{1}{2^{n-1}}$

9. Which of the following statement is not true?
(a) Between two integers, there exist infinite number of rational numbers
(b) Between two rational numbers, there exist infinite number of integers
(c) Between two rational numbers, there exist infinite number of rational numbers
(d) Between two real numbers, there exists infinite number of real numbers

10. Four rational numbers between 3 and 4 are:
(a) $\frac{3}{5},\frac{4}{5},1,\frac{6}{5}$ (b) $\frac{13}{5},\frac{14}{5},\frac{16}{5},\frac{17}{5}$
(c) 3.1, 3.2, 4.1, 4.2 (d) 3.1, 3.2, 3.8, 3.9

11. If 2009 = $p^a.q^b$, where "p" and "q" are prime number, then find the value of p + q. **[NTSE]**
(a) 3 (b) 48
(c) 51 (d) 2009

12. When expanded, the number of zeroes in 1000^{10} is: **[NTSE]**
(a) 13 (b) 30
(c) 4 (d) 10

13. If $2^{x-1}+2^{x+1} = 320$, then the value of x is- **[NTSE]**
(a) 6 (b) 8
(c) 5 (d) 7

14. If a and b are positive integers less than 10 such that a^b = 125, then $(a-b)^{a+b-4}$ is equal to: **[NTSE]**
(a) 16 (b) 25
(c) 28 (d) 30

15. If $x = \frac{1}{1+\sqrt{2}}$, then the value of $x^2 + 2x + 3$ is **[NTSE]**
(a) 3 (b) 0
(c) 4 (d) 1

16. If $1^3 + 2^3 + ... + 9^3 = 2025$, then $(0.11)^3 + (0.22)^3 + ...(0.99)^3$ will be- **[NTSE]**
(a) 0.2695 (b) 2.695
(c) 3.695 (d) 0.3695

17. The value of $\frac{\sqrt{4^5}+\left(\sqrt{2}\right)^{10}}{\left(\sqrt[3]{4}\right)^9-\left(\sqrt[3]{2}\right)^9}\times\sqrt{9}$ is **[JSTSE]**
(a) $\frac{8}{7}$ (b) $\frac{15}{7}$
(c) $\frac{18}{7}$ (d) $\frac{24}{7}$

18. If $9^{x-2} = 3^{x+1}$, then the value of 2^{1+x} is **[JSTSE]**
(a) 64 (b) 32
(c) 16 (d) 5

19. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}-2}{\sqrt{3}+2}$, then the value of $x^2+\left(\frac{39}{x}\right)^2$ is **[JSTSE]**
(a) 104 (b) 114
(c) 124 (d) 144

20. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals **[JSTSE]**

(a) $5\sqrt{5}$ (b) $\sqrt{5}$

(c) $25\sqrt{5}$ (d) 125

21. Simplify: $\dfrac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}}$ **[Olympiad]**

(a) 1 (b) 0

(c) $\dfrac{5}{3}$ (d) $-\dfrac{5}{3}$

More Than One Option Correct

DIRECTIONS : *This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct.*

1. Which of the following is irrational?

(a) $\sqrt{17}$ (b) $\dfrac{\sqrt{12}}{\sqrt{3}}$

(c) $\sqrt{7}$ (d) $\sqrt{81}$

2. Value of $\sqrt[4]{(81)^{-2}}$ is

(a) $\dfrac{1}{9}$ (b) $\dfrac{1}{3}$

(c) 9 (d) 3

3. Which of the following is equal to x ?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$

(c) $(\sqrt{x^3})^{\frac{2}{3}}$ (d) $x^{12/19} + x^{7/19}$

4. Which of the following is/are not correct ?

(a) Every whole number is a natural number.

(b) Every integer is a rational number

(c) Every rational number is an integer

(d) Every rational number is a whole number

5. Which of the following is/are correct ?

(a) There are infinitely many rational numbers between any two given rational numbers.

(b) Every point on the number line represents a unique real number.

(c) The decimal expansion of an irrational number is non-terminating non-recurring.

(d) A number whose decimal expansion is non-terminating non-recurring is rational.

Passage/Case Based Questions

DIRECTIONS : *Study the given paragraph(s) and answer the following questions.*

Passage-I

Case II: One day the teacher wrote laws of exponents on the black board and the sheets containing questions based on these Laws were distributed among the students to conduct a open book test.

The laws written on the black board are as follow:

$x^m \cdot x^n = x^{m+n}$

$(x^m)^n = x^{mn}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$x^0 = 1$

$x^{-n} = \dfrac{1}{x^n}, x \neq 0$

With given conditions of $x > 0$ be real number and m, n be rational numbers.

1. The value of $\sqrt[6]{(729)^{-1}}$ is

(a) 3 (b) $\dfrac{1}{3^{-2}}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{9}$

2. $3(a^4b^3)^{10} \times 5(a^2b^2)^3$ is equal to

(a) $15a^{36}b^{36}$ (b) $15a^{46}b^{46}$

(c) $15a^{56}b^{46}$ (d) $15a^{46}b^{36}$

3. If $2^{x+1} = 4^{x-3}$, then the value of x is

(a) 7 (b) 6 (c) 5 (d) 8

4. $5^{2/3} \times 5^{2/6}$ is equal to

(a) $5^{2/9}$ (b) $5^{1/3}$ (c) 5 (d) 1

5. The value of expression $\left(\dfrac{x^a}{x^b}\right)^c \times \left(\dfrac{x^b}{x^c}\right)^a \times \left(\dfrac{x^c}{x^a}\right)$ is

(a) x^{a+b+c} (b) x^{a-b-c}

(c) $\dfrac{1}{x}$ (d) 1

Passage-II

When the denominator of an expression contains term with a square root. The procedure of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator. For example

(i) $(\sqrt{a} + \sqrt{b})$ is the rationalising factor of $(\sqrt{a} - \sqrt{b})$ and vice versa.

(ii) $a + \sqrt{b}$ is the rationalising factor of $a - \sqrt{b}$ and vice versa.

6. The numerator of $\dfrac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} + \dfrac{a - \sqrt{a^2 - b^2}}{a + \sqrt{a^2 + b^2}}$

(a) a^2 (b) b^2 (c) $a^2 - b^2$ (d) $4a^2 - 2b^2$

7. If $x = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ and $y = \dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, find $x^2 + y^2$.

(a) 98 (b) 80 (c) 89 (d) 90

8. Rationalising factor of $5 + 2\sqrt{6}$ is

(a) $5 + 2\sqrt{6}$ (b) $5 - 2\sqrt{6}$

(c) $5 + \sqrt{6}$ (d) None of these

Assertion & Reason Questions

DIRECTIONS : *Each of these questions contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.
(b) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.
(c) If **Assertion** is **correct** but **Reason** is **incorrect.**
(d) If **Assertion** is **incorrect** but **Reason** is **correct.**

1. **Assertion :** Every integer is a rational number
Reason : Every integer 'm' can be expressed in the form $\frac{m}{1}$.

2. **Assertion :** $\sqrt{2}$ is an irrational number.
Reason : A number is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

3. **Assertion :** $17^2 . 17^5 = 17^3$
Reason : If $a > 0$ be a real number and p and q be rational numbers. Then $a^p . a^q = a^{p+q}$.

4. **Assertion :** $2+\sqrt{6}$ is an irrational number.
Reason : Sum of a rational number and an irrational number is always an irrational number.

5. **Assertion :** A rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{5}{12}$.
Reason : Rational number between two numbers a and b is $\sqrt{ab}$

Numeric/Integer Type Questions

DIRECTIONS : *Answer the following questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9.*

1. If $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a+b\sqrt{7}$, find the product of a and b.

2. If $x=\frac{\sqrt{p+2q}+\sqrt{p-2q}}{\sqrt{p+2q}-\sqrt{p-2q}}$ and $q \neq 0$, then find qx^2-px+q

3. If $x=9+4\sqrt{5}$ and $xy=1$, then $\frac{1}{322}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$ is

4. The value of x, if $5^{x-3}.3^{2x-8}=225$ is

5. If $a^2bc^3=25$ and $ab^2=5$, then abc equals

6. If $x+1/x=3$, then x^2+1/x^2 is equal to

SOLUTIONS

Brief Explanations of Selected Questions

Let's Revise Through FIB & T/F

1. rational
2. infinite
3. real
4. False
5. False
6. True
7. False
8. False
9. True
10. 2
11. conjugate
12. 9
13. irrational
14. True
15. True
16. True
17. True
18. $\sqrt{\dfrac{6}{35}}$
19. $\left(\dfrac{1008}{63}\right)^{1/4} = (16)^{1/4} = (2^4)^{1/4} = 2$
20. irrational

EXERCISE-1 Master Board

Multiple Choice Questions

1. **(b)** Let x = 0.12333...... ...(i)

Multiplying by 10 on both sides, we get

10x = 1.2333..... ...(ii)

Subtracting (i) from (ii), we get

$$9x = 1.11 \Rightarrow x = \frac{111}{900}$$

2. **(c)** $$x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5})^2-2^2}}{\sqrt{5}+1}$$

$$= \frac{2\sqrt{5}+2}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{\sqrt{5}+1} = 2$$

3. **(a)** $$\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}} = 7^m \Rightarrow \left[\{(7^{-2})^{-2}\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \left[(7^4)^{-1/3}\right]^{\frac{1}{4}} = 7^m \Rightarrow (7^{-4/3})^{1/4} = 7^m \; 7^{-1/3} = 7^m$$

$\therefore m = -1/3$

4. **(c)** Irrational number.

5. **(b)** Let x = 0.12333...... ...(i)

Multiplying by10 on both sides, we get

10x = 1.2333..... ...(ii)

Subtracting (i) from (ii), we get

$$9x = 1.11 \Rightarrow x = \frac{111}{900}$$

6. **(d)** Rational or irrational

7. **(b)** $\dfrac{21}{3}, \dfrac{24}{3}$

8. **(d)** Given : $x = 2 + \sqrt{3}, xy = 1$

$$\frac{x}{2-x} + \frac{y}{2-y}$$

$$= \frac{x}{2-x} + \frac{xy}{2x-xy} = \frac{x}{2-x} + \frac{1}{2x-1}$$

$$= \frac{2+\sqrt{3}}{2-\left(2+\sqrt{3}\right)} + \frac{1}{2\left(2+\sqrt{3}\right)-1}$$

$$= \frac{-(2+\sqrt{3})}{\sqrt{3}} + \frac{1}{2\left(2+\sqrt{3}\right)-1}$$

$$= \frac{-\left(2\sqrt{3}+3\right)}{3} + \frac{1}{3+2\sqrt{3}} = \frac{-\left(3+2\sqrt{3}\right)^2+3}{3\left(3+2\sqrt{3}\right)}$$

$$= \frac{-\left(9+12+12\sqrt{3}\right)+3}{3\left(3+2\sqrt{3}\right)}$$

$$= \frac{-\left(18+12\sqrt{3}\right)}{3\left(3+2\sqrt{3}\right)} = \frac{-6\left(3+2\sqrt{3}\right)}{3\left(3+2\sqrt{3}\right)} = -2$$

9. **(d)** Rationalising each term, we get

$$\frac{1}{1+\sqrt{2}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$$

$$\frac{1}{\sqrt{3}+\sqrt{4}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$\frac{1}{\sqrt{15}+\sqrt{16}} = \frac{1}{4+\sqrt{15}} \times \frac{4-\sqrt{15}}{4-\sqrt{15}}$$

$$= \frac{4-\sqrt{15}}{16-15} = 4-\sqrt{15}$$

$\therefore$ Given expression

$$= \left(\sqrt{2}-1\right) + \left(\sqrt{3}-\sqrt{2}\right) + \left(2-\sqrt{3}\right) + \ldots. + 4 - \sqrt{15}$$

$= 4 - 1 = 3$

10. **(a)** $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$$=\frac{2+\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{4+3+4\sqrt{3}}{4-3}+\frac{4+3-4\sqrt{3}}{4-3}+\frac{3+1-2\sqrt{3}}{3-1}$$

$$=\frac{7+4\sqrt{3}}{1}+\frac{7-4\sqrt{3}}{1}+\frac{4-2\sqrt{3}}{2}$$

$$=7+4\sqrt{3}+7-4\sqrt{3}+2-\sqrt{3}$$

$$=16-\sqrt{3}=16-1.732=14.268$$

11. **(a)** $x-\frac{1}{x}=\sqrt{3}$ ....(i)

$$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+x\times\frac{1}{x}\right)$$

$$=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right) \qquad \text{....(ii)}$$

squaring both sides of (i)

$$\left(x-\frac{1}{x}\right)^2=\left(\sqrt{3}\right)^2$$

$$x^2+\frac{1}{x^2}-2.x.\frac{1}{x}=3$$

$$\Rightarrow x^2+\frac{1}{x^2}-2=3$$

$$\Rightarrow x^2+\frac{1}{x^2}=5$$

Substitute in (ii) to get

$$x^3-\frac{1}{x^3}=\sqrt{3}\left(5+1\right)=6\sqrt{3}$$

12. **(c)** Let $x=2.74\overline{35}$

$100x=274.\overline{35}$ ....(i)

$10000\,x=27435.35$ ....(ii)

subtracting (i) from (ii)

$9900x=27161$

$x=\frac{27161}{9900}$.

Assertion & Reason Questions

1. **(c)** $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{5}{12}$

2. **(a)** Both assertion and reason are true. Reason is the correct explanation of assertion.

3. **(b)** **Reason:** If possible, let the sum of a rational number A and an irrational number $\sqrt{b}$ be a rational number c.

Then, $a+\sqrt{b}=c \Rightarrow \sqrt{b}=(c-a)$

But, the difference of two rationals is a rational.

So, $(c-a)$ is rational and therefore, $\sqrt{b}$ is rational.

Thus, we arrive at a contradiction.

So, our supposition is wrong.

Hence, the sum of a rational and an irrational is irrational.

Assertion: If possible , let $\sqrt{3}$ be rational and its simplest form is $\frac{a}{b}$

Then, $\sqrt{3}-\frac{a}{b} \Rightarrow \frac{a^2}{b^2}-3 \Rightarrow -3\frac{a^2}{b}-3b$

Clearly, 3b is an integer and $\frac{a^2}{b}$ is not an integer since $(a, b)=1$.

Thus, we arrive at a contradiction.

Hence, $\sqrt{3}$ is an irrational number.

Clearly, Reason does not give Assertion. So, (b) holds.

Very Short Answer Questions

1. 0 **2.** –1 **3.** a

4. (i) $-1.\overline{3}$ (ii) $0.91\overline{6}$ (iii) $0.\overline{538461}$

5. (i) rational (ii) rational

6. No, for example $\sqrt{4}=2$ is a rational number.

7. (i) 3^{-21} (ii) $56^{1/2}$

8. (i) $2^{\frac{3}{4}}$ (ii) $\frac{125}{216}$

(iii) $\frac{729}{125}$ (iv) $\frac{30375}{16}$

(v) 2^4

9. 0.10101001000100001 and 0.11001001000100001

10. (i) terminating (ii) repeating

(iii) repeating (iv) terminating

11. 37/1100 **12.** 0.25 and 0.2525

13. $\sqrt[3]{4}$ is the greatest **14.** $\frac{7}{24},\frac{13}{48},\frac{15}{48},\frac{31}{96}$

Short Answer Questions

1. $p=-11/2$, $q=-5/2$ **2.** 5

3. (i) irrational (ii) rational

4. $\frac{6}{3\sqrt{2}-2\sqrt{3}}=\frac{6}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$$= \frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2} = \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$$

$$= \frac{6(3\sqrt{2}+2\sqrt{3})}{6} = 3\sqrt{2}+2\sqrt{3}$$

Therefore, $3\sqrt{2}+2\sqrt{3} = 3\sqrt{2}-a\sqrt{3}$

$\Rightarrow \quad a = -2$

5. $$\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left((2^3)^{\frac{1}{3}}+(3^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$$

$$= \left[5(2+3)^3\right]^{\frac{1}{4}} = \left[5(5)^3\right]^{\frac{1}{4}} = \left[5^4\right]^{\frac{1}{4}} = 5$$

6. On rationalizing the denominator, we get

$$\frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})}$$

$$= \frac{(2+\sqrt{2})(3-\sqrt{5})}{(3+\sqrt{5})(2-\sqrt{2})} \times \frac{(3-\sqrt{5})(2+\sqrt{2})}{(3-\sqrt{5})(2+\sqrt{2})}$$

$$= \frac{(2+\sqrt{2})^2(3-\sqrt{5})^2}{(3^2-(\sqrt{5})^2)(2^2-(\sqrt{2})^2)} = \frac{(4+2+4\sqrt{2})(9+5-6\sqrt{5})}{(9-5)(4-2)}$$

$$= \frac{(6+4\sqrt{2})(14-6\sqrt{5})}{4\times 2} = \frac{84-36\sqrt{5}+56\sqrt{2}-24\sqrt{10}}{8}$$

Hence irrational.

7. $$\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192} = 5\sqrt{3}+\frac{4}{2}\sqrt{3}-8\sqrt{3}$$

$$= \sqrt{3}(5+2-8) = -1.732$$

8. $$\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \times \frac{x-\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}-y} = \frac{x^2-\left(\sqrt{x^2-y^2}\right)^2}{\left(\sqrt{x^2+y^2}\right)^2-y^2}$$

$$= \frac{x^2-(x^2-y^2)}{(x^2+y^2)-y^2} = \frac{y^2}{x^2}$$

9. $$x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5})^2-2^2}}{\sqrt{5}+1}$$

$$= \frac{2\sqrt{5}+2}{\sqrt{5}+1} = \frac{2(\sqrt{5}+1)}{\sqrt{5}+1} = 2$$

10. Consider a unit square OABC onto the number line with the vertex O which coincides with zero.

Then OB $= \sqrt{1^2+1^2} = \sqrt{2}$

Construct BD of unit length perpendicular to OB. Then

OD $= \sqrt{(\sqrt{2})^2+1^2} = \sqrt{3}$

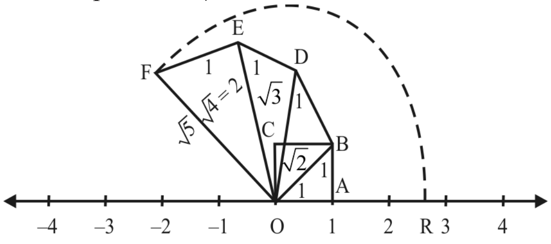
Construct DE of unit length perpendicular to OD. Then

OE $= \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$

Similarly, construct EF of unit length perpendicular to OE.

Then OF $= \sqrt{2^2+1^2} = \sqrt{5}$

Using a compass, with centre O and radius OF, draw an arc which intersects the number line in the point R. Then R corresponds to $\sqrt{5}$.



Representation of $\sqrt{5}$

Alternately: Students can also visualise $\sqrt{5} = \sqrt{(2)^2+(1)^2}$ there by taking base as 2 units and perpendicular as 1 unit

11. We have $$\frac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4}$$

$$= \frac{x^{2(a+b)}x^{2(b+c)}x^{2(c+a)}}{(x^a)^4.(x^b)^4(x^c)^4}$$

$$= \frac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}} = \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}} = \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}} = 1$$

12. Let $x = 3\sqrt{3}+\sqrt{26}$

$$\frac{1}{x} = \frac{1}{3\sqrt{3}+\sqrt{26}} \times \frac{3\sqrt{3}-\sqrt{26}}{3\sqrt{3}-\sqrt{26}}$$

$$= \frac{3\sqrt{3}-\sqrt{26}}{(27)-(26)} = 3\sqrt{3}-\sqrt{26}$$

$$\therefore \quad \frac{1}{2}\left(x+\frac{1}{x}\right) = \frac{1}{2}\left[(3\sqrt{3}+\sqrt{26})+(3\sqrt{3}-\sqrt{26})\right]$$

$$= \frac{1}{2}\times 6\sqrt{3} = 3\sqrt{3}$$

13. $$\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{\frac{-1}{3}}\right]^{\frac{1}{4}} = 7^m \Rightarrow \left[\{(7^{-2})^{-2}\}^{-1/3}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \quad \left[(7^4)^{-1/3}\right]^{\frac{1}{4}} = 7^m$$

$$\Rightarrow \quad (7^{-4/3})^{1/4} = 7^m \Rightarrow \quad 7^{-1/3} = 7^m$$

$$\therefore \quad m = -1/3$$

14. The two numbers 0.1201001000100001....... and 0.12101001000100001........ are the irrational numbers between 0.12 and 0.13.

15. One rational number between $\frac{1}{5}$ and $\frac{7}{10}$

$= \frac{1}{2}\left(\frac{1}{5}+\frac{7}{10}\right) = \frac{1}{2}\left(\frac{2+7}{10}\right) = \frac{9}{20}$

Second rational number between $\frac{1}{5}$

and $\frac{7}{10} = \frac{1}{2}\left(\frac{1}{5}+\frac{9}{20}\right) = \frac{1}{2}\left(\frac{4+9}{20}\right) = \frac{13}{40}$

Third rational number between $\frac{1}{5}$

and $\frac{7}{10} = \frac{1}{2}\left(\frac{13}{40}+\frac{1}{5}\right) = \frac{1}{2}\left(\frac{13+8}{40}\right) = \frac{21}{80}$

16. Let $x = 1.272727$.......... Since two digits are repeating, we multiply x by 100 to get

$100x = 127.2727$ ....

So, $100x = 126 + 1.272727......... = 126 + x$

Therefore, $100x - x = 126, \Rightarrow 99x = 126 \Rightarrow x = \frac{126}{99} = \frac{14}{11}$

17. Here, $x = \sqrt{2}-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} \quad \text{(By Rationalising)}$$

$$= \frac{\sqrt{2}+\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} \quad [\because (a-b)(a+b) = (a^2-b^2)]$$

$$= \frac{\sqrt{2}+\sqrt{3}}{2-3}$$

$$\frac{1}{x} = \frac{\sqrt{2}+\sqrt{3}}{-1} = -(\sqrt{2}+\sqrt{3})$$

Now, $x + \frac{1}{x} = (\sqrt{2}-\sqrt{3}) - (\sqrt{2}+\sqrt{3})$

$$= \sqrt{2}-\sqrt{3}-\sqrt{2}-\sqrt{3} = -2\sqrt{3}$$

18. Given, $6^x = 30$

Now, $6^{x-1} + 6^{x+1}$

$= 6^x \cdot 6^{-1} + 6^x \cdot 6$

$= 6^x\left(\frac{1}{6}\right) + 6^x . 6$

$= 30\left(\frac{1}{6}\right) + 30(6)$

$= 5 + 180 = 185$

Long Answer Questions

1. $0.6 + 0.\overline{7} + 0.4\overline{7}$

Let $x = 0.\overline{7}\,;\, y = 0.4\overline{7}$

$\Rightarrow \quad 9x = 7.\overline{7}\,;\, 10y = 4.\overline{7}$

$\Rightarrow \quad 9x = 7\,;\, 100y = 47.\overline{7}$

$\Rightarrow \quad x = \frac{7}{9}\,;\, 90y = 43 \Rightarrow y = \frac{43}{90}$

$\therefore$ Required expression $= \frac{6}{10} + \frac{7}{9} + \frac{43}{90}$

2. $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{1}$

$\Rightarrow x = 5 + 2\sqrt{6}$

also it can be observed that $x.y = 1$.

$\therefore \; y = 5 - 2\sqrt{6}$

$\therefore x^2 + y^2 = (5+2\sqrt{6})^2 + (5-2\sqrt{6})^2$

$= 49 + 49 = 98$

3. $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$= \sqrt{10}+\sqrt{4\times5}+\sqrt{4\times10}-\sqrt{5}-\sqrt{16\times5}$

$= \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$

$= 3\sqrt{10}-3\sqrt{5} = 3(\sqrt{10}-\sqrt{5})$

$\therefore$ Given expression

$$= \frac{15}{3(\sqrt{10}-\sqrt{5})} = \frac{5}{\sqrt{10}-\sqrt{5}} = \frac{5}{\sqrt{10}-\sqrt{5}} \times \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}+\sqrt{5}}$$

$$= \frac{5(\sqrt{10}+\sqrt{5})}{10-5} = \sqrt{10}+\sqrt{5}$$

4. Let I $= \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = A - B - C$

where $A = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{10-3}$

$$= \frac{7\sqrt{30}-7\times3}{7} = \frac{7(\sqrt{30}-3)}{7} = \sqrt{30}-3$$

$$B = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{30}-2\times5}{6-5} = 2\sqrt{30}-10$$

$$C = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \frac{3\sqrt{30}-18}{15-18} = \frac{3\sqrt{30}-18}{-3}$$

$= -\sqrt{30}+6$

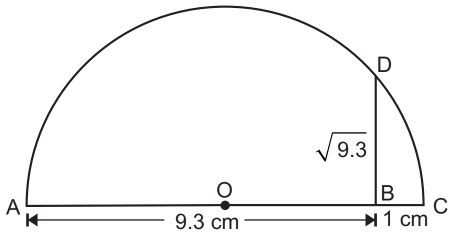
Now, $I = A - B - C$

$= (\sqrt{30}-3)-(2\sqrt{30}-10)-(-\sqrt{30}+6)$

$= \sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6$

$= 2\sqrt{30}-2\sqrt{30}-3+10-6 = 1$

5.



Mark a distance 9.3 units from a fixed point A on a given line to obtain a given point B such that AB = 9.3 units. From B mark a distance of 1 unit and call the new point as C. Find the mid point of AC and call that point as O. Draw a semi circle with centre O and radius OC = 5.15 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D.

Then, BD = $\sqrt{9.3}$ = 3.05 units.

6. Let $x = 2.\overline{6}$ ...(i)

$\therefore\ 10x = 26.\overline{6}$ ...(ii)

Subtracting (i) from (ii)

$9x = 24$

$\therefore\ x = \frac{24}{9} = \frac{8}{3}$

Also suppose $y = 0.\overline{9}$ ...(iii)

$\Rightarrow y = 0.9\overline{9}$

$\therefore\ 10y = 9.\overline{9}$ ...(iv)

Subtracting equation (iii) from (iv), we get $9y = 9$

$\therefore\ y = \frac{9}{9} = 1$

$\therefore\ 2.\overline{6} - 0.\overline{9} = x - y = \frac{8}{3} - 1 = \frac{8-3}{3} = \frac{5}{3}$

7. $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{3}-\sqrt{6}}{\sqrt{3}-\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{3-6} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$$

$= -\sqrt{2}(\sqrt{3}-\sqrt{6}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) - \sqrt{6}(\sqrt{2}-\sqrt{3})$

$= -\sqrt{6}+\sqrt{12}-\sqrt{18}+\sqrt{6}-\sqrt{12}+\sqrt{18} = 0$

8. As $x = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$ (on rationalizing)

$\Rightarrow\ x-2 = -\sqrt{3}$

Squaring both sides, we get

$(x-2)^2 = (-\sqrt{3})^2 \Rightarrow x^2+4-4x = 3 \Rightarrow x^2-4x+1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x^3 - 4x^2 + x + 3x^2 - 12x + 3$

$x(x^2 - 4x + 1) + 3(x^2 - 4x + 1) = x \times 0 + 3(0) = 0 + 0$

$= 0$

9. Let $x = 0.12\overline{3}$ i.e. $x = 0.12333$ ... (i)

Multiply both sides of (i) by 100

$100x = 12.333$ ... (ii)

Multiply both sides of (ii) by 10

$1000x = 123.333$ ... (iii)

Subtract (ii) from (iii)

$1000x = 123.333$

$100x = 12.333$

$900x = 111.0$

$\Rightarrow\ x = \frac{111}{900} = \frac{3\times37}{900} = \frac{37}{300}$

10. Given, $9^{a+1} = 81^{b+2}$

$\Rightarrow (3)^{2(a+1)} = (9)^{2(b+2)}$

$\Rightarrow 3^{2a+2} = (3)^{2\times2(b+2)}$

$\Rightarrow 3^{2a+2} = 3^{4b+8}$

Equating powers on both sides, we get

$2a + 2 = 4b + 8$

$\Rightarrow 2a - 4b = 6$

$\Rightarrow 2(a - 2b) = 6$

$\Rightarrow a - 2b = 3$

$\Rightarrow a = 3 + 2b$ ....(i)

Also, $\left(\frac{1}{3}\right)^{3+a} = \left(\frac{1}{27}\right)^{3b}$

$\Rightarrow \left(\frac{1}{3}\right)^{3+a} = \left(\frac{1}{3}\right)^{3\times3b}$

$\Rightarrow \left(\frac{1}{3}\right)^{3+a} = \left(\frac{1}{3}\right)^{9b}$

Equating powers on both sides, we get

$3 + a = 9b$

$\Rightarrow a - 9b = (-3)$

$\Rightarrow 3 + 2b - 9b = (-3)$ [From (i)]

$\Rightarrow 3 - 7b = (-3)$

$\Rightarrow 3 + 3 = 7b$

$$\frac{6}{7} = b$$

and $a = 3 + 2b$ [From (i)]

$$= 3 + 2\left(\frac{6}{7}\right) = 3 + \frac{12}{7} = \frac{21+12}{7} = \frac{33}{7}$$

11. Given expression is

$$\frac{16 \times 2^{x+1} - 4 \times 2^{x}}{16 \times 2^{x+2} - 2 \times 2^{x+2}}$$

$$= \frac{2^4 \times 2^{x+1} - 2^2 \times 2^x}{2^4 \times 2^{x+2} - 2 \times 2^{x+2}}$$

$$= \frac{2^4 \times 2^x \times 2 - 2^2 \times 2^x}{2^4 \times 2^x \times 2^2 - 2 \times 2^x \times 2^2}$$

$$= \frac{2^5 \times 2^x - 2^2 \times 2^x}{2^6 \times 2^x - 2^3 \times 2^x}$$

$$= \frac{2^2 \times 2^x \left(2^3 - 1\right)}{2^3 \times 2^x \left(2^3 - 1\right)}$$

$$= \frac{2^2}{2^3} = 2^{2-3} = 2^{-1} = \frac{1}{2}$$

(Hence proved)

12. $\left(\sqrt{323 + 1 + 2\sqrt{323}}\right) - \left(\sqrt{323 + 1 - 2\sqrt{323}}\right)$

$$= \left(\sqrt{\left(\sqrt{323}\right)^2 + (1)^2 + 2\sqrt{323}}\right)^2$$

$$- \left(\sqrt{\left(\sqrt{323}\right)^2 + (1)^2 - 2(1)\left(\sqrt{323}\right)}\right)$$

$$\left[\begin{array}{l} \because a^2 + b^2 + 2ab = (a+b)^2 \\ \text{and } a^2 + b^2 - 2ab = (a-b)^2 \end{array}\right]$$

$$= \left(\sqrt{\left(\sqrt{323} + 1\right)^2}\right) - \left(\sqrt{\left(\sqrt{323} - 1\right)^2}\right)$$

$$= \left(\sqrt{323} + 1\right) - \left(\sqrt{323} - 1\right)$$

$$= \sqrt{323} + 1 - \sqrt{323} + 1 = 2$$

HOTS Questions

1. Consider the rational number $\frac{5}{7}$. On dividing by 7, we get,

```
7 ) 5.000000 (0.714285.......
    49
     10
      7
      30
      28
       20
       14
        60
        56
         40
         35
          5
```

Thus, $\frac{5}{7} = 0.714285 \ldots\ldots = 0.\overline{714285}$

Now, consider $\frac{9}{11}$.

On dividing by 11, we get

```
11) 9.0000 (0.8181......
    88
     20
     11
      90
      88
       20
       11
        9
```

Thus, $\frac{9}{11} = 0.8181\ldots\ldots = 0.\overline{81}$

Three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$ can be

0.75 075007500075000075.......,

0.7670767000767............ and

0.808008000800008...........

2. Let $x = 2.5434343$

$\Rightarrow x = 2.5\overline{43}$

Multiplying both sides by 10 we get

$10x = 25.\overline{43}$ … (i)

Again multiplying equation (i) by 100

$1000x = 2543.\overline{43}$ … (ii)

On subtracting equation (i) from (ii)

$1000x - 10x = 2543.\overline{43} - 25.\overline{43}$

$990x = 2518$

$$x = \frac{2518}{990} \Rightarrow x = \frac{1259}{495}$$

Hence, $2.54343... = \frac{1259}{495}$

3. Given $x = \frac{1}{7+4\sqrt{3}}$

on rationalising,

we get $x = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = 7-4\sqrt{3}$

$\because\ y = \frac{1}{7-4\sqrt{3}} \Rightarrow x.y = 1$

and also $y = 7+4\sqrt{3}$

for $5x^2 - 5y^2 - 7xy$.

$$= 5\left[(7-4\sqrt{3})^2 - \left(7+4\sqrt{3}\right)^2\right] - 7\times 1$$

$$= 5\left[49+48-56\sqrt{3}-49-48-56\sqrt{3}\right]-7$$

$$= 5\left[-112\sqrt{3}\right]-7 \ = -7\left[1+80\sqrt{3}\right]$$

EXERCISE-2 NCERT Questions

Text-book Questions

1. Yes! zero is a rational number. We can write zero in the form $\frac{p}{q}$, as follows :

$0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ ......so on., q can be negative integer also.

2. $\frac{3}{5} = \frac{3\times 10}{5\times 10} = \frac{30}{50}$, $\frac{4}{5} = \frac{4\times 10}{5\times 10} = \frac{40}{50}$, therefore,

five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$.

3. (i) True, since the collection of whole numbers contains all natural numbers.

(ii) False, because, -3 is not a whole number.

(iii) False, $\because\ \frac{1}{2}$ is not a whole number.

4. (i) True, ($\because$ real numbers are collection of rational and irrational numbers.)

(ii) False; 0 is not a natural number.

(iii) False, (2 is real but not irrational.)

5. No, the square roots of all positive integers are not irrational. For example, $\sqrt{16} = 4$ is a rational number.

6. Consider $125^{1/3} = (5^3)^{1/3} = 5^{3\times 1/3} = 5^1 = 5$.

7. (i) Consider $32^{2/5} = (2^5)^{2/5} = 2^{5\times 2/5} = 2^2 = 4$.

(ii) Consider $125^{-1/3} = (5^3)^{-1/3} = 5^{3\times(-1/3)} = 5^{-1} = \frac{1}{5}$

8. (i) Consider $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{2/3+1/5}$

$= 2^{\frac{10+3}{15}} = 2^{13/15}$.

(ii) Consider $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} = 3^{-21}$

(iii) Consider $\frac{11^{1/2}}{11^{1/4}} = 11^{1/2-1/4} = 11^{1/4}$.

Exemplar Questions

1.	**(c)**	**2.**	**(c)**
3.	**(d)**	**4.**	**(d)**
5.	**(d)**	**6.**	**(c)**
7.	**(d)**	**8.**	**(c)**
9.	**(c)**	**10.**	**(c)**

11. (i) $3\sqrt{3} + 2\sqrt{3\times 3\times 3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 2\sqrt{3\times 3\times 3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 6\sqrt{3} + \frac{7}{\sqrt{3}}$

$= \frac{3\sqrt{3}\times\sqrt{3} + 6\sqrt{3}\times\sqrt{3}+7}{\sqrt{3}}$

$= \frac{9+18+7}{\sqrt{3}} = \frac{34}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{34\sqrt{3}}{3}$

(ii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$= \sqrt[4]{3\times 3\times 3\times 3} - 8\sqrt[3]{6\times 6\times 6}$

$+ 15\sqrt[5]{2\times 2\times 2\times 2\times 2} + \sqrt{15\times 15}$

$= 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15$

$= -45 + 45$

$= 0$

12. $\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$

$$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}}$$

$$= 4\left(6^3\right)^{\frac{2}{3}} + \left(4^4\right)^{\frac{3}{4}} + 2\left(3^5\right)^{\frac{1}{5}}$$

$$= 4 \times 6^{\frac{6}{3}} + 4^{\frac{12}{4}} + 2 \times 3^{\frac{5}{5}} = 4 \times 6^2 + 4^3 + 2 \times 3$$

$$= 4 \times 36 + 64 + 6 = 144 + 64 + 6 = 214$$

EXERCISE-3 **Foundation Builder**

Multiple Choice Questions

1. **(a)** $2^{x+4} . 3^{x+1} = 2^5.3^2 \Rightarrow x = 1$

2. **(a)** **3. (d)**

4. **(a)** $x = 6$ satisfies the given equation

5. **(d)** Simplest form of $\frac{17}{119} = \frac{17}{119}$

Simplest form of $\frac{14}{57} = \frac{14}{57}$

Simplest form of $\frac{21}{38} = \frac{21}{38}$

Simplest form of $\frac{21}{57} = \frac{7}{19}$

6. **(a)** **7.** **(a)** **8.** **(c)** **9.** **(b)**

10. **(d)** From the given options (a) and (b) does not contain rational number between 3 and 5.

(c) has 4.1 and 4.2 that does not lie between 3 and 4.

11. **(b)** The prime factorization of, 2009 is,

$2009 = 7 \times 7 \times 41 = 7^2 \times 41$

Compare with $2009 = p^a.q^b$, we get

$p = 7, q = 41$

$\Rightarrow p + q = 7 + 41 = 48$

12. **(b)** Total number of zeroes in the number 1000 is 3

Also exponent of the number 1000 is 10

Therefore, the total number of zeroes are

$3 \times 10 = 30$.

13. **(d)** $2^{x-1} + 2^{x+1} = 320$

$$\frac{2^x}{x} + 2^x \times 2 = 320$$

$$2^x \times \left(\frac{1}{2} + 2\right) = 320$$

$$2^x \times \frac{5}{2} = 320$$

$2^x = 128 = 2^7 \Rightarrow x = 7$

14. **(a)** $a^b = 125$

We know that $5^3 = 125$

So, $a = 5$, $b = 3$

Now, the value of $(a - b)^{a+b-4}$ is

$(5 - 3)^{5+3-4} = 2^4 = 16$

15. **(c)** $x = \frac{1}{1+\sqrt{2}}$

after rationalization

$$x = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$x = \sqrt{2} - 1$

so $x^2 = 3 - 2\sqrt{2}$

$\therefore x^2 + 2x + 3 \Rightarrow (3 - 2\sqrt{2}) + 2(\sqrt{2} - 1) + 3 = 4$

16. **(b)** $1^3 + 2^3 + ... 9^3 = 2025$

$(0.11)^3 + (0.22)^3 + ...+ (0.99)^3$

$\Rightarrow (0.11)^3 [1^3 + 2^3 + ... 9^3]$

$\Rightarrow 0.001331 \times 2025 = 2.695275$

17. **(d)**
$$\frac{\sqrt{4^5} + \left(\sqrt{2}\right)^{10}}{\left(\sqrt[3]{4}\right)^9 - \left(\sqrt[3]{2}\right)^9} \times \sqrt{9} = \frac{\sqrt{2^{10}} + \left(\sqrt{2}\right)^{10}}{(4)^{\frac{9}{3}} - (2)^{\frac{9}{3}}} \times 3$$

$$= \frac{2^5 + 2^5}{4^3 - 2^3} \times 3$$

$$= \frac{64}{56} \times 3 = \frac{8}{7} \times 3 = \frac{24}{7}$$

18. **(a)** $9^{x-2} = 3^{x+1} \Rightarrow 3^{2x-4} = 3^{x+1}$

$\Rightarrow 2x - 4 = x + 1 \Rightarrow x = 5$

$\therefore 2^{1+x} = 2^{1+5} = 2^6 = 64$

19. **(b)**
$$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}-2}{\sqrt{3}+2}$$

$$= \frac{10\sqrt{3}+12}{4+2\sqrt{3}} = \frac{5\sqrt{3}+6}{2+\sqrt{3}} = 4\sqrt{3} - 3$$

$$\therefore x^2 + \left(\frac{39}{x}\right)^2 = \left(4\sqrt{3} - 3\right)^2 + \left(\frac{39}{4\sqrt{3}-3}\right)^2$$

$$= 57 - 24\sqrt{3} + \frac{507}{19 - 8\sqrt{3}}$$

$$= 57 - 24\sqrt{3} + 3(19 + 8\sqrt{3}) = 57 + 57 = 114$$

20. **(c)** $4^x - 4^{x-1} = 24 \Rightarrow 4^x - 4^x \,.\, 4^{-1} = 24$

$$\Rightarrow 4^x\left(1 - \frac{1}{4}\right) = 24 \Rightarrow 4^x\left(\frac{3}{4}\right) = 24$$

$$\Rightarrow 4^x = 24 \times \frac{4}{3} \Rightarrow 4^x = 32 \Rightarrow 2^{2x} = 2^5$$

$$\Rightarrow x = \frac{5}{2}$$

$$\therefore\ (2x)^x = \left(2 \times \frac{5}{2}\right)^{\frac{5}{2}} = 5^{\frac{5}{2}} = \sqrt{5 \times 5 \times 5 \times 5 \times 5}$$

$$= 25\sqrt{5}$$

21. **(d)** $\dfrac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}} = \dfrac{5^n[25 - 30]}{5^n[13 - 10]} = \dfrac{-5}{3}$

More Than One Option Correct

1. **(a, c)**
2. **(b)**
3. **(c)**
4. **(a, c, d)**
5. **(a, b, c)**

Passage/Case Based Questions

1. **(a)** We have, $\sqrt[6]{(729)^{-1}} = (3^{-6})^{\frac{1}{6}} = 3^{-6 \times \frac{1}{6}}$

$$= 3^{-1} = \frac{1}{3}$$

2. **(d)** We have, $3(a^4b^3)^{10} \times 5(a^2b^2)^3$

$3 \times 5 \times a^{4\times10} \times b^{3\times10} \times a^{2\times3} \times b^{2\times3}$ $(\because (x^m)^n) = x^{mn})$

$15a^{40}b^{30}a^6b^6 = 15a^{40+6}\, b^{30+6}$ $(\because x^m.^n = x^{m+n})$

$= 15a^{46}b^{36}$

3. **(a)** We have, $2^{x+1} = 4^{x-3}$

$2^{x+1} = 2^{2x-6} \Rightarrow x + 1 = 2x - 6 \Rightarrow x = 7$

4. **(c)** We have, $5^{2/3} \times 5^{2/6} = 5^{2/3 + 1/3} = 5^{3/3} = 5$

5. **(d)** We have, $\left(\dfrac{x^a}{x^b}\right)^c \times \left(\dfrac{x^b}{x^c}\right)^a \times \left(\dfrac{x^c}{x^a}\right)^b$

$$= x^{(ac-bc)} \times x^{(ab-ac)} \times x^{(bc-ab)}$$

$$= x^{(ac-ac+bc-bc+ca-ca)} = x^0 = 1$$

6. **(d)**

7. **(a)**

8. **(b)**

Assertion & Reason Questions

1. **(a)** **2.** **(a)**

3. **(d)** $17^2 . 17^5 = 17^{2+5} = 17^7$

4. **(a)**

5. **(c)** $\dfrac{1}{2}\left(\dfrac{1}{3} + \dfrac{1}{2}\right) = \dfrac{5}{12}$

Numeric/Integer Type Questions

1. **Ans : 0**

$$\text{L.H.S.} = \frac{\sqrt{7} - 1}{\sqrt{7} + 1} - \frac{\sqrt{7} + 1}{\sqrt{7} - 1} = \frac{(\sqrt{7} - 1)^2 - (\sqrt{7} + 1)^2}{(\sqrt{7} + 1)(\sqrt{7} - 1)}$$

$$= \frac{(7 + 1 - 2\sqrt{7}) - (7 + 1 + 2\sqrt{7})}{(\sqrt{7})^2 - (1)^2}$$

$$= \frac{8 - 2\sqrt{7} - 8 - 2\sqrt{7}}{7 - 1} = -\frac{4\sqrt{7}}{6} = -\frac{2}{3}\sqrt{7}$$

$$\therefore\quad -\frac{2}{3}\sqrt{7} = a + b\sqrt{7}$$

$$\Rightarrow\quad a = 0 \text{ and } b = -\frac{2}{3}$$

$$\Rightarrow\quad a \times b = 0$$

2. **Ans : 0**

$$\frac{x}{1} = \frac{\sqrt{p + 2q} + \sqrt{p - 2q}}{\sqrt{p + 2q} - \sqrt{p - 2q}}$$

$$\frac{x + 1}{x - 1} = \frac{2\sqrt{p + 2q}}{2\sqrt{p - 2q}} \quad \text{(By componendo and dividendo)}$$

On squaring both sides, we get

$$\frac{(x^2 + 1) + 2x}{(x^2 + 1) - 2x} = \frac{p + 2q}{p - 2q}$$

On again applying componendo and dividendo.

we have,

$$\frac{2(x^2 + 1)}{2.2x} = \frac{2.p}{2.2q}$$

$$qx^2 + q = px$$

Then, $qx^2 - px + q = 0$.

3. **Ans : 1**

$$x = 9 + 4\sqrt{5}$$

$$y = \frac{1}{x} = \frac{1}{9 + 4\sqrt{5}} = \frac{9 - 4\sqrt{5}}{(9)^2 - (4\sqrt{5})^2} = 9 - 4\sqrt{5}$$

$\therefore \quad \frac{1}{322}\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$

$= \frac{1}{322}[(9-4\sqrt{5})^2+(9+4\sqrt{5})^2]$

$= \frac{2(81+80)}{322} = \frac{2(161)}{322} = \frac{322}{322} = 1$

4. Ans : 5

$5^{x-3}\,3^{2x-8} = 225 \Rightarrow 5^{x-3}\,3^{2x-8} = 5^2.3^2$

By comparing $x-3=2 \Rightarrow x=3+2=5.$

5. Ans : 5

$a^2bc^3 = 25$ ......(i)

$ab^2 = 5$ ......(ii)

Multiplying (i) & (ii), we get $a^3\,b^3\,c^3 = 125$

$abc = (125)^{1/3} = 5$

6. Ans : 7

$x+\frac{1}{x}=3$ (given)

Squaring both sides $x^2+\frac{1}{x^2}+2=9$

$\Rightarrow \quad x^2+\frac{1}{x^2}=7.$